FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
OPEN TEST – I
PAPER –1
TEST DATE: 09-02-2025

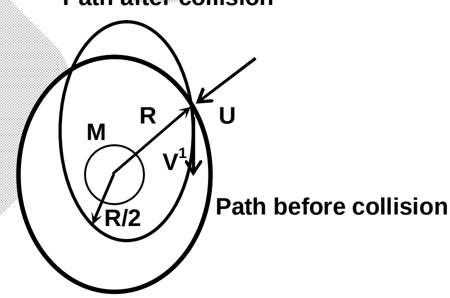
ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. C

Sol. Situation before and after collision is shown in figure.



Before collision the speed of satellite is $U_0 = \sqrt{\frac{GM}{R}}$

Consider the meteorite collide with speed u and after collision the combined mass 11m moves at an angle θ with the orbit.

Apply LCM $mu = 11mV^1 \sin\theta$ -----(1)

and $10mU_0 = 11mV^1 \cos\theta$ -------(2)

Apply LCAM, $11m(V^1 \cos\theta)R = 11mV_p(R/2)$ ----(3) here V_p is speed at perigee.

Apply LCE, $-\frac{GM11m}{R} + \frac{1}{2}11mV^{1^2} = -\frac{GM11m}{R/2} + \frac{1}{2}11mV_p^2$ ----(4)

Solving $U = \sqrt{\frac{58GM}{R}}$

2. B

Sol. $W_{ext} = \left(\frac{\sigma}{2\varepsilon_0}\right)(\lambda\ell)\left(\frac{\ell}{2}\right)\cos 180° = -\frac{\lambda\sigma_0\ell^2}{4\varepsilon_0}$ (Motion is under constant force and point of application of force will be at mid point of line charges during the process)

$V_A - V_B = \frac{\sigma\ell}{2\varepsilon_0}$

3. B

Sol. $Z_1 = R_1 + X_L i = 8 + 6i$

$Z_2 = R_2 - X_C i = 3 - 4i$

Now, $\frac{1}{Z'} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$\frac{1}{Z'} = \frac{1}{(8+6i)} + \frac{1}{(3-4i)}$$
$$= \frac{(8-6i)}{100} + \frac{(3+4i)}{25} = \frac{20+10i}{100}$$
$$\frac{1}{Z'} = \frac{(2+i)}{10} \Rightarrow Z' = \frac{10}{(2+i)}$$
$$Z' = 2(2-i)$$
$$Z' = (4-2i) \quad \ldots(i)$$
$$Z_3 = R_3 + X_L' i = (2+10i) \quad \ldots(ii)$$
Hence, net impedance of the circuit is
$$Z = Z' + Z_3 = (4-2i) + (2+10i)$$
Z = 6 + 8i
$$|Z| = \sqrt{(6)^2 + (8)^2} = 10\ \Omega$$
Hence, $I = \frac{90}{|Z|} = \frac{90}{10} = 9$ amp

4. C

Sol. $E_1 = \frac{hC}{\lambda_1} = \frac{1240}{310} = 4\text{ eV}$

$E_2 = \frac{hC}{\lambda_2} = \frac{1240}{400} = 3.1\text{ eV}$

Also, $\frac{k_1}{k_2} = 4 \Rightarrow k_1 = 4k_2$

Now, $E_1 = k_1 + \phi$

$k_1 + \phi = 4$ …(i)

$E_2 = k_2 + \phi$

$k_2 + \phi = 3.1$ …(ii)

Solving (i) and (ii), we get

$k_1 - k_2 = 0.9$

$4k_2 - k_2 = 0.9$

$3k_2 = 0.9$

$\Rightarrow k_2 = 0.3$ eV

$k_1 = 1.2$ eV

from equation (i),

$k_1 + \phi = 4$

$\phi = 4 - k_1 = 4 - 1.2 = 2.8$ eV

$\phi = 2.8$ eV

5. A, C, D

Sol. At open end pressure amplitude is zero while at closed end pressure amplitude is maximum.

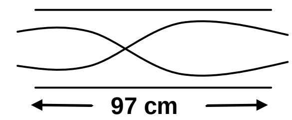
6. A, C

Sol. zero error = – 1.25 mm

Reading = 18 + 0.34 + 1.25 = 19.59 mm

7. A, B, C, D

Sol. (A) $\frac{\lambda}{2} = 97 + 0.6D$ …(i)



$v = f\lambda$

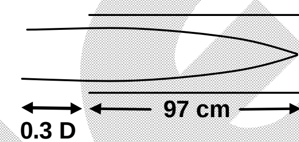
$\lambda = \frac{320}{160} = 2m = 200\,cm$ …(ii)

From (i) and (ii)

$\frac{200}{2} = 97 + 0.6D$

D = 5 cm

(B) $\frac{\lambda}{4} = 97 + 0.3D$

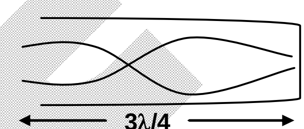


$\lambda = (97 \times 4 + 1.2D)$

$\lambda = (97 \times 4 + 1.2 \times 5) = 394\,cm$

$f = \frac{v}{\lambda} = \frac{32000}{394} = 81.22\,Hz$

(C) $\frac{3\lambda}{4} = 97 + 0.3D$



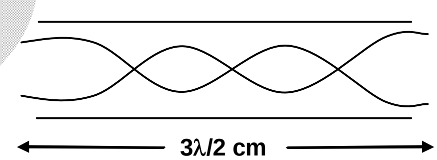
$= 97 + (0.3)(5)$

= 98.5

$\lambda = (98.5)\left(\frac{14}{3}\right) = \frac{394}{3}$

$f = \frac{v}{\lambda} = \frac{32000 \times 3}{394} = 243.65\,Hz$

(D) $\frac{5\lambda}{2} = 97 + 0.6D$



$f = \frac{v}{\lambda} = \frac{320 \times 5 \times 100}{200} = 800$ Hz

8. A

Sol. (P) $z_1 = 10 + 10i = 10(1 + i)$

$Z_2 = 20 - 20i = 20(1 - i)$

$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{10(1+i)} + \frac{1}{20(1-i)}$

$\frac{1}{z} = \left(\frac{1-i}{20}\right) + \left(\frac{1+i}{40}\right) = \left(\frac{3-i}{40}\right)$

$z = \frac{40}{(3-i)} \times \frac{(3+i)}{(3+i)} = 4(3+i)$

$I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{40\sqrt{2}}{4\sqrt{10}} = 2\sqrt{5}$ amp

(Q) $z_1 = 20(1 - i)$

$Z_2 = 20(1 + i)$

$\frac{1}{z'} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{20(1-i)} + \frac{1}{20(1+i)}$

$\frac{1}{z'} = \frac{(1+i)}{40} + \frac{(1-i)}{40} = \frac{1}{20}$

$z' = 20\ \Omega$
$z = 20 + 15i$
$\Rightarrow I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{25\sqrt{2}}{25} = \sqrt{2}$ amp

(R) $x = (x_L - x_C) = 40 - 20 = 20\,\Omega$
$\frac{1}{z'} = \frac{1}{20} + \frac{1}{20i} = \frac{1-i}{20}$
$z' = \frac{20}{1-i} = 10(1+i)$
$z = 10 + 10(1+i) = 10(2+i)$
$\Rightarrow I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{20\sqrt{10}}{10\sqrt{5}} = 2\sqrt{2}$ amp

(S) $\frac{1}{z'} = \frac{1}{10(1+i)} + \frac{1}{10(1-i)} = \frac{(1-i)}{20} + \frac{(1+i)}{20} = \frac{1}{10}$
$z' = 10\Omega$
$\Rightarrow z = 10 + (20 + 40i) = 30 + 40i$
$I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{100\sqrt{10}}{50} = 2\sqrt{10}$ amp

9. D

Sol. $A \to B$
TP^2 = constant
$PV^{1/3} =$ constant
$W_{A\to B} = \frac{P_1V_1 - P_2V_2}{x-1} = \frac{nR(T_1 - T_2)}{x-1} = -\frac{9}{2}nRT_0$
$\Delta U_{A\to B} = -3nRT_0$
$\Delta Q_{A\to B} = -\frac{15}{2}nRT_0$
$C \to A$
$TP^4 =$ constant
$PV^{1/5} =$ constant
$W_{C\to A} = \frac{nR(T_1 - T_2)}{x-1} = \frac{15}{4}nRT_0$
$\Delta U_{C\to A} = nR(T_A - T_C) > 0$
$\Delta Q_{C\to A} > 0$
$B \to C$
$\Delta U = 0$
$W_{B\to C} > 0$
$\Delta Q_{B\to C} > 0$

10. B

Sol. $\tau = \frac{1}{\frac{6\times 12}{6+12} + \frac{18\times 24}{18+24}} = 0.07$
$\Rightarrow$ At steady state

$R_{eq} = \frac{25}{2}$

Current through battery = 2 ampere
Current through inductor = 1/6 ampere
$\Rightarrow$ At one time constant

Current through inductor = $0.63 \times \frac{1}{6} = 0.105$ ampere

From KVL, current through 12Ω = 1.353 ampere
$\Rightarrow$ For part (iv) after switching

$\frac{1}{6}L = i_L(L + 2L)$

$i_L = \frac{1}{18} = 0.056$ ampere

11. C

Sol. (P) $v_I = \frac{4}{3} \times 3 = 4$

$v_r = 4 - 3 = 1$ m/s

(Q) $\frac{dv}{dt} = -\left(\frac{v}{u}\right)^2 \frac{du}{dt}$

$\Rightarrow v_I = 1$ m/s and $v_r = 10$ m/s

(R) $v_{I,m} = 4$

$\Rightarrow v_{I,g} = 7 \Rightarrow v_r = 8$ m/s

(S) final image is at the position of object hence magnification is 1

$\Rightarrow v_I = v_0 = 1$m/s

$\Rightarrow v_r = 0$

SECTION – B

12. 3

Sol. In case I:

$P - P_0 = \frac{4T}{r}$

Now radius 'r' increases to '3r' due to charge on the soap bubble

$P_1V_1 = P_2V_2$

$P\frac{4}{3}\pi r^3 = P_2\frac{4}{3}\pi(3r)^3$

$P_2 = \frac{P}{27}$

In case II:

$P_2 + \frac{\sigma_1^2}{2\varepsilon_0} - P_0 = \frac{4T}{3r}$, where σ_1 is final charge density ($\sigma_1 = \sigma/9$)

$\frac{P}{27} + \frac{\sigma^2}{162\varepsilon_0} - P_0 = \frac{P - P_0}{3}$

$\therefore\ P = \left(\frac{\sigma^2}{48\varepsilon_0} - \frac{9}{4}P_0\right)$

13. 8

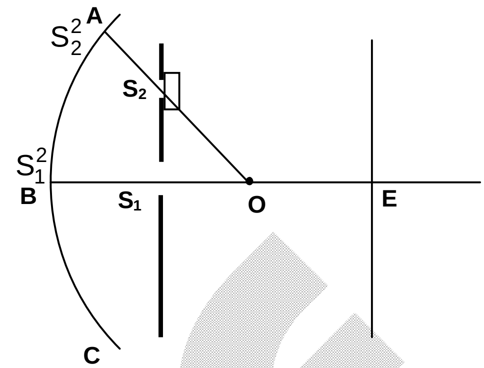
Sol. Let S_1^1 and S_2^1 are the points on the wave front where perpendiculars can be drawn from S_1 and S_2. For central fringe formed at E path difference should be zero.

$$S_2^1S_2 + (\mu - 1)t + S_2E = S_1^1S_1 + S_1E$$

$$(OS_2^1 - OS_2) + (\mu - 1)t + \sqrt{D^2 + d^2} = (OS_1^1 - OS_1) + D$$

$$(\mu - 1)t = (OS_2 - OS_1) - (\sqrt{D^2 + d^2} - D)$$

using binomial approximation $t = \frac{31\lambda}{8}$

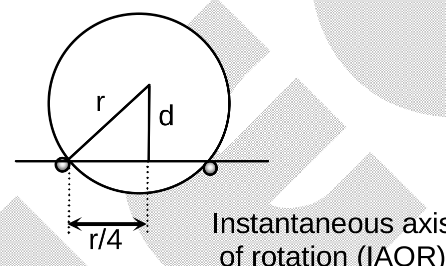


14. 5

Sol. $d = \frac{\sqrt{15}}{4}r$

$v = d\omega$

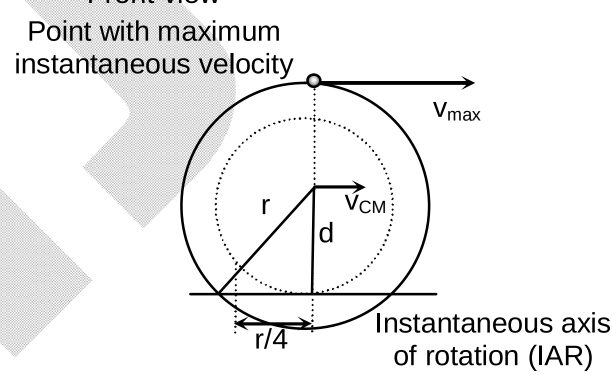
$\omega = \frac{4v}{\sqrt{15}r}$



Front view

$v_{max} = \omega(r + d)$

$v_{max} = \left(\frac{15 + 4\sqrt{15}}{15}\right)v$



Side view

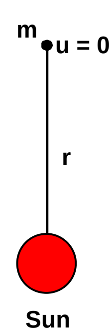
15. 4

Sol. $a = \frac{r + 0}{2} = \frac{r}{2}$

Using Kepler's law

$\frac{T}{T_0} = \left(\frac{r/2}{r}\right)^{3/2}$

$T = \frac{T_0}{2\sqrt{2}}$



The time taken by the body to fall on the surface of sun,

$\tau = \frac{T}{2} = \frac{T_0}{4\sqrt{2}}$

$\tau = \frac{T_0}{4\sqrt{2}}$

Hence, n = 4

16. 480

Sol. Power obtained = 1200 mega watt

$= 1200 \times 10^6 \times 3600 = 432 \times 10^{10}$ J

The output energy from the power house

$$E = \frac{(432 \times 10^{10}) \times 100}{20} = 216 \times 10^{11}\,J$$

Let this energy is obtained from Δm kg

$\Delta mc^2 = 216 \times 10^{11=}$

$$\Delta m = \frac{216 \times 10^{11}}{9 \times 10^{16}} = 24 \times 10^{-5}\,kg$$

Hence the uranium required

$$m = \frac{3 \times 24 \times 10^{-5}}{1.5 \times 10^{-3}} = 48 \times 10^{-2}\,kg$$

m = 480 g

17. 7

Sol. $P_{in} = \frac{4S}{R} + \frac{4S}{2R} = \frac{6S}{R}$

$P_{mid} = \frac{4S}{2R} = \frac{2S}{R}$

as T = constant

$$\frac{P_{in}}{P_{mid}} \frac{V_{in}}{V_{mid}} = \frac{\mu_{in}}{\mu_{mid}} = \frac{3}{7} = y$$

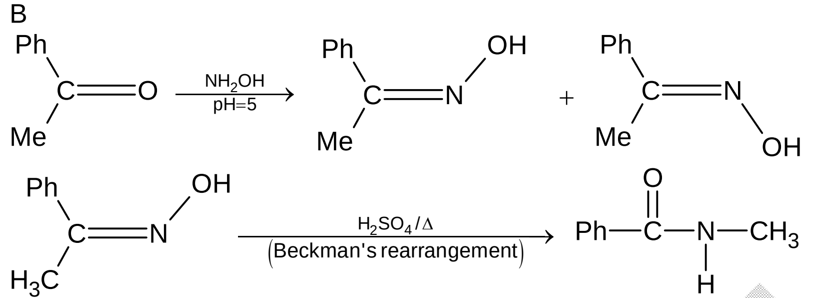
and $\frac{P_{in}}{P_{mid}} = \frac{\rho_{in}}{\rho_{mid}} = 3 = x$

Chemistry

PART – II

SECTION – A

18. B

Sol.

N – methyl benzamide

19. B

Sol.

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$$

Initial mol	1	1	0
mol at eqbm	$(1-x)$	$(1-x)$	$2x$
eqbm conc	$\frac{(1-x)}{V}$	$\frac{(1-x)}{V}$	$\frac{2x}{V}$

Now, equivalents of hypo used = equivalents of I_2 present at equilibrium

$0.1(10\times1) = (1-x)\times 2$

x = 0.5

$$\text{So, } K_c = \frac{(1)^2}{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)} = 4$$

So, K_c for the reaction

$$HI(g) \rightleftharpoons \frac{1}{2}H_2(g) + \frac{1}{2}I_2(g) \text{ will be } \left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1}{2} = 0.5$$

Hence, (B)

20. A

Sol. (A) is yellow while (B), (C) and (D) all are blue.

21. B

Sol. A minimum of 6-H-atoms should be present in the sample, so that all the 10 spectral lines can be observed.

22. A, C, D

Sol. (A) $\Delta S_{X\to Z} = \frac{q}{T} = 0 \text{(reversible adiabatic change)}$

(B) $\Delta S_{X\to Z\to Y} = \Delta S_{X\to Y} = 2.303\times5\times8.314\log_{10}\frac{100}{10} = 95.7\ JK^{-1}$

(because 'S' is a state function).

(C) $\Delta S_{X\to Y} = 95.7\ JK^{-1}\text{(Reversible isothermal expansion)}$

(D) Since 'S' is a state function, so $\Delta S_{X\to Y} = \Delta S_{X\to Z} + \Delta S_{Z\to Y} = 95.7\ JK^{-1}$

$= 0 + \Delta S_{z\to Y} = 95.7$

So, A, C and D are correct.

23. A, B, C

Sol. (i)

	PO_4^{3-}	$+ H^+ \longrightarrow$	HPO_4^{2-}
m.mol.	10	5	0
after neutralization	5	0	5

So, $pH = pK_{a_3} = 13$ (Buffer)

(ii)

	PO_4^{3-}	$+ H^+ \longrightarrow$	HPO_4^{2-}
m.mol.	10	10	0
after neutralization	0	0	10

So, $pH = \frac{8+13}{2} = 10.5$ (Amphiprotic anion)

(iii)

	PO_4^{3-}	$+ H^+ \longrightarrow$	HPO_4^{2-}
m.mol.	10	15	0
after neutralization	0	5	10

HPO_4^{2-}	$+ H^+ \longrightarrow$	$H_2PO_4^-$
10	5	0
5	0	5

So, $pH = pK_{a_2} = 8$ (Buffer)

(iv)

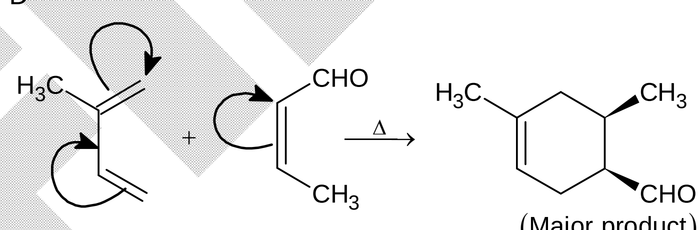
	PO_4^{3-}	$+ H^+ \longrightarrow$	HPO_4^{2-}
m.mol.	10	20	0
after neutralization	0	10	10

	HPO_4^{2-}	$+ H^+ \longrightarrow$	$H_2PO_4^-$
m.mol.	10	10	0
after neutralization	0	0	10

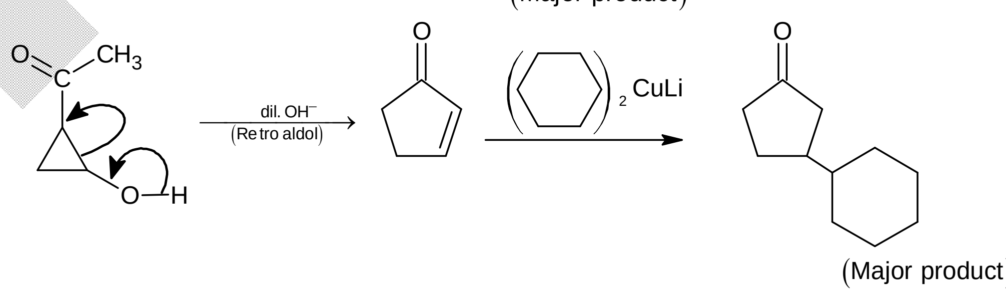
So, $pH = pK_{a_3} = 5.5$ (Amphiprotic anion)

24. B, C, D

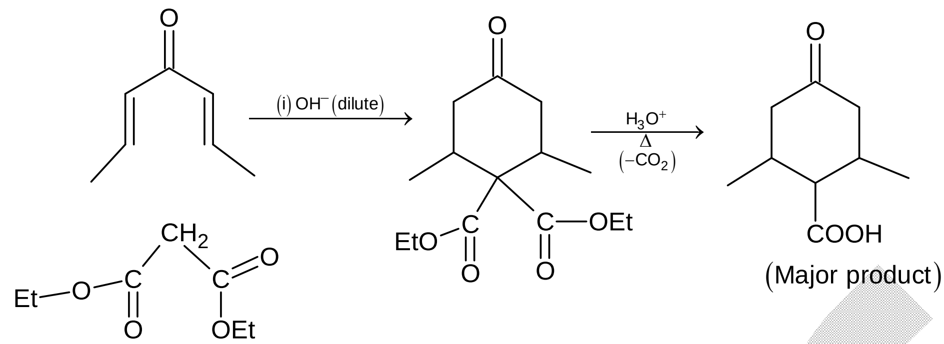
Sol. (A)



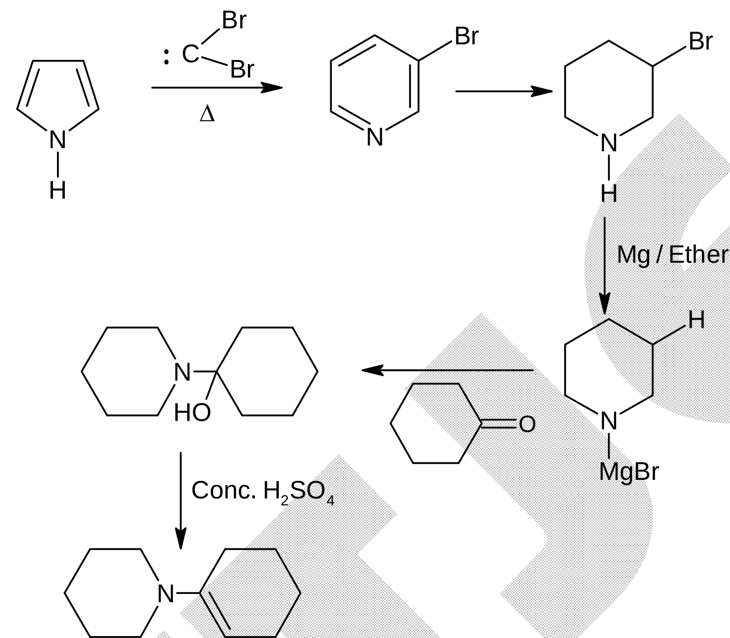
(B)



(C)



(D)



(Major product)

25. B

Sol. $\underline{P}_4 + O_2(\text{Excess}) \longrightarrow \underline{P}_4O_{10} \xrightarrow{6PCl_5} \underline{P}OCl_3$

$\underline{P}_4(\text{Excess}) + O_2 \longrightarrow P_4O_6 \xrightarrow{6PCl_5} \underline{P}OCl_3 + \underline{P}Cl_3$

$\underline{P}_4 + \text{Conc. } HNO_3 \longrightarrow H_3\underline{P}O_4 \xrightarrow{3PCl_5} \underline{P}OCl_3 + HCl$

$P_4(\text{White}) + \underline{S}O_2Cl_2 \longrightarrow \underline{S}O_2 + PCl_5$

$\downarrow PCl_5$

$\underline{S}OCl_2 + POCl_3$

26. C

Sol. $CuSO_4 + 4NH_3(aq.) \longrightarrow [Cu(NH_3)_4]^{2+} \rightarrow$ Blue, sp^2d, $\mu = \sqrt{3}$ BM; paramagnetic

$CoCl_2 + KNO_2 + CH_3COOH \xrightarrow{pH>7} [Co(NO_2)_6]^{3-} \rightarrow$ yellow, diamagnetic, (d^2sp^3), $\mu = 0$

$CoCl_2 + NH_4SCN \longrightarrow [Co(SCN)_4]^{2-} \rightarrow$ blue, paramagnetic, $\mu = 3.87$ BM, sp^3

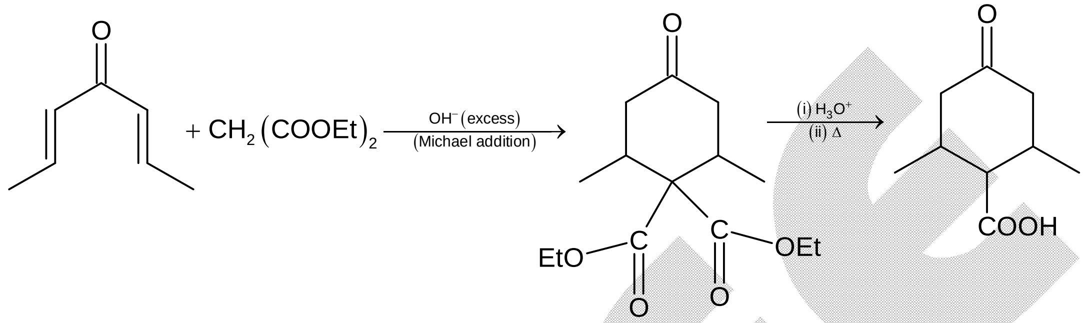
$$CuSO_4 + \underset{(excess)}{KCN} \longrightarrow \left[Cu(CN)_4\right]^{3-} + (CN)_2$$

$$\downarrow$$

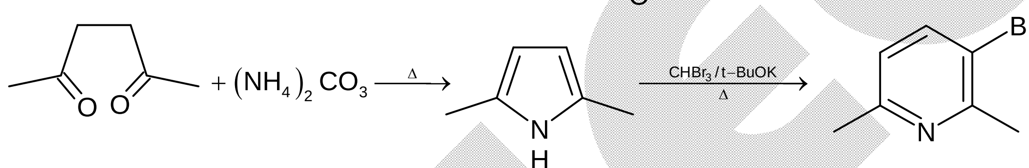
sp^3, colourless, $\mu = 0$, diamagnetic

27. A

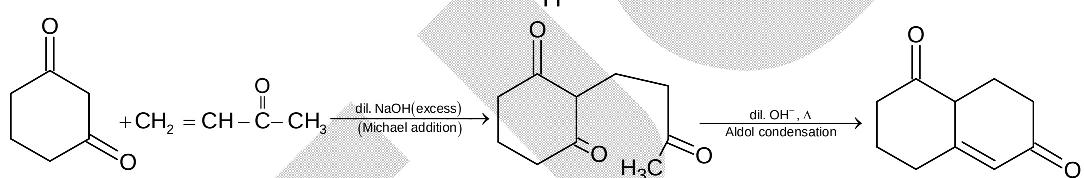
Sol. (P)



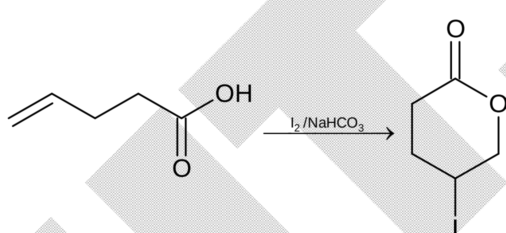
(Q)



(R)



(S)



28. B

Sol.
$$K_2Cr_2O_7 + 7H_2O_2 + 4KOH \longrightarrow \underset{\text{(red brown solid)}}{2K_3CrO_8} + 9H_2O$$

$$K_2Cr_2O_7 + H_2O_2 + H_2SO_4 \longrightarrow \underset{\text{(blue)}}{2CrO_5} + K_2SO_4 + 5H_2O$$

$$3K_4\left[Fe(CN)_6\right] + \underset{\text{(dil.)}}{6H_2SO_4} \xrightarrow{\Delta} \underset{\text{(triatomioc gas)}}{12HCN} \uparrow + Fe_2\left[Fe(CN)_6\right] + 6K_2SO_4$$

$$2KMnO_4 + \text{Conc. } 2H_2SO_4\ (\text{cold}) \longrightarrow \underset{\text{(green oily)}}{Mn_2O_7} + 2KHSO_4 + H_2O$$

SECTION – B

29. 10

Sol. Possible isomers are

$K_3\left[Fe(CN)_6\right]$ $K_3\left[Fe(NC)_6\right]$

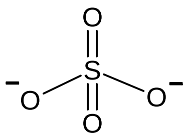
$K_3\left[Fe(CN)_5(NC)\right]$ $K_3\left[Fe(NC)_5(CN)\right]$

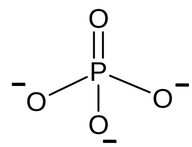
$K_3\left[\underset{\text{cis and trans}}{Fe(CN)_4(NC)_2}\right]$ $K_3\left[\underset{\text{cis and trans}}{Fe(NC)_4(CN)_2}\right]$

$K_3\left[\underset{\text{fac and mer}}{Fe(CN)_3(NC)_3}\right]$

30. 3

Sol.





Number of resonating structure of $SO_4^{2-} = 6$

Number of plane of symmetry in $SO_4^{2-} = 6$

Number of resonating structure of $PO_4^{3-} = 4$

So, $\frac{6+6}{4} = 3$

31. 0

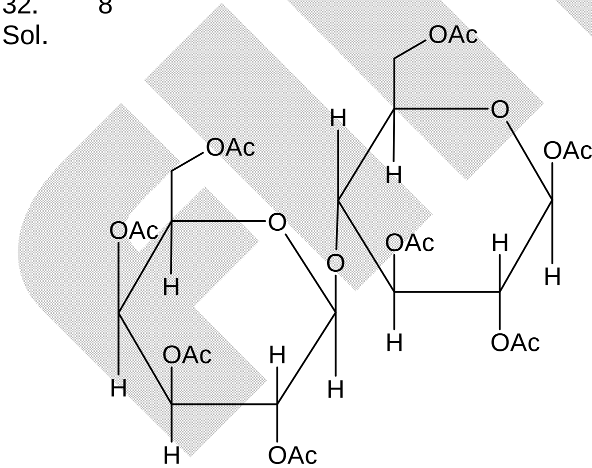
Sol.

	Species	Geometry	Hybridization	d-orbitals involved
I.	$[VO(acac)_2]$	Square pyramidal	dsp^3	$d_{x^2-y^2}$
II.	$[Fe(CO)_5]$	Trigonal bi-pyramidal	dsp^3	d_{z^2}
III.	$[PtCl_4]^{2-}$	Square planar	dsp^2	$d_{x^2-y^2}$
IV.	$K_4[Fe(CN)_6]$	Octahedral	d^2sp^3	$d_{x^2-y^2}$, d_{z^2}
V.	$[IF_7]$	Pentagonal bi-pyramidal	sp^3d^3	$d_{x^2-y^2}$, d_{z^2} and d_{xy}

So, x = 3, y = 4 and z = 1

32. 8

Sol.



33. 5

Sol.

$RH_2 + Ca^{2+} \longrightarrow RCa + 2H^+$

3×10^{-4} M

So, $\left[Ca^{2+}\right] = \frac{3}{2} \times 10^{-4}$ M

So, mass of $CaCO_3$ present in 1 L hard water $= \frac{3}{2} \times 10^{-4} \times 100 = 1.5 \times 10^{-2}$ g

So, degree of hardness of water $= \frac{1.5 \times 10^{-2}}{1500} \times 10^{6} = 10$ g

So, degree of hardness = 10 ppm.
So, 2x = 10 and x = 5.

34. 5
Sol. Only, reaction II, IV, V, VI, VII will give tert-butyl benzene as the major product.

Mathematics

PART – III

SECTION – A

35. D

Sol. $A^n = \begin{bmatrix} 2^n & 3^n - 2^n \\ 0 & 3^n \end{bmatrix}$

$A^{14} = \begin{bmatrix} 2^{14} & 3^{14} - 2^{14} \\ 0 & 3^{14} \end{bmatrix}$

a + d = $4^7 + 9^7$, which is multiple of 13

36. B

Sol. $g(x) = e^{-x^2} f(x)$ is concave downward and $e^{-x^2} f(x) > x$

$\Rightarrow f(x) > xe^{x^2} \Rightarrow f(x) > \frac{xe^{x^2}}{e}$ also, $f\left(\frac{1}{2}\right) > \frac{1}{2e}$

37. B

Sol. Let $\frac{a_1}{a_1+1} = \frac{a_2}{a_2+3} = ... = \frac{a_{1013}}{a_{1013}+2025} = \frac{1}{k}$

$\Rightarrow a_1 = \frac{1}{k-1}, a_2 = \frac{3}{k-1}, ..., a_{1013} = \frac{2025}{k-1}$

$\therefore$ a_1, a_2, a_3 ..... are in A.P.

$\Rightarrow$ Also, $a_1 + a_2 + ... + a_{1013} = \frac{(1013)^2}{k-1} = 2026$

$\Rightarrow \frac{1}{k-1} = \frac{2}{1013} \Rightarrow a_{41} = \frac{81}{k-1} = \frac{162}{1013}$

38. C

Sol. Let f(x) = x^2 + ax + b ; g(x) = x^2 + cx + d

From given conditions 104a + 3b = 104c + 3d

$\Rightarrow \frac{d-b}{a-c} = \frac{104}{3}$

Now, g(x) = f(x) $\Rightarrow x = \frac{b-d}{c-a} = \frac{104}{3}$

39. A, B, C, D

Sol. $I = \sum_{k=0}^{\infty} \int_0^{2^n} 2^k \left[\frac{x}{2^k}\right] dx = \sum_{k=0}^{n-1} 2^k \int_0^{2^n} \left[\frac{x}{2^k}\right] dx$, $n \in N$

$= \sum_{k=0}^{n-1} 2^k \int_0^{2^n} \left[\frac{2^n - x}{2^k}\right] dx = \sum_{k=0}^{n-1} 2^k \int_0^{2^n} \left(\frac{2^n}{2^k} - 1 - \left[\frac{x}{2^n}\right]\right) dx = \sum_{k=0}^{n-1} (2^n - 2^k) 2^n - I$

$\Rightarrow 2I = n \cdot 2^{2n} - 2^n \left(\frac{2^n - 1}{2-1}\right) = (n-1)2^{2n} + 2^n$

40. A, B, C

Sol. $\because$ p(x) has 2024 roots in $(-\infty, 1)$
$\therefore$ Using Rolle's theorem $p'(x)$ has also all roots in $(-\infty, 1)$
Now, $f'(x) = p(e^x)$ and $f''(x) = p'(e^x)e^x$ but $e^x > 1\ \forall\ x \in (0, \infty)$
$\therefore$ $f'(x)$ and $f''(x)$ have no real roots in $(0, \infty)$

41. A, B, C, D

Sol. Let $\int_0^1 g(\alpha)d\alpha = k\ \ \therefore f(\alpha) = \alpha - k$

Now, $g(t) = 1 + \frac{t^3}{2} - t\int_0^t (x-k)dx = 1 + \frac{t^3}{2} - t\left(\frac{t^2}{2} - kt\right)$

$g(t) = 1 + kt^2$

Now, $k = \int_0^1 (1 + k\alpha^2)d\alpha = 1 + \frac{k}{3} \Rightarrow k = \frac{3}{2}$

$\Rightarrow f(x) = x - \frac{3}{2}$

42. D

Sol. (P) A = I + B, where $B = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$
Now, $B^2 = B^3 = \ldots\ldots 0$
$\Rightarrow A^{2024} = (I + B)^{2024} = I + 2024B$
trace$(A^{2024}) = 2$

(Q) For infinite solution: $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \beta \\ 5 & 1 & 2 \end{vmatrix} = 0 \Rightarrow \beta = 0$$

$$D_1 = \begin{vmatrix} \alpha & 4 & -1 \\ -3 & 9 & 0 \\ -1 & 1 & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1$$

$\Rightarrow 4\beta - 3\alpha = 3$

(R) Vectors are coplanar

$$\begin{vmatrix} \cos A & 1 & 1 \\ 1 & \cos B & 1 \\ 1 & 1 & \cos C \end{vmatrix} = 0 \Rightarrow \operatorname{cosec}^2\frac{A}{2} + \operatorname{cosec}^2\frac{B}{2} + \operatorname{cosec}^2\frac{C}{2} = 2$$

(S) Let required plane is $(x + y + z + 1) + \lambda(2y + z - 4) = 0$
Direction ratio of above plane: $1,\ 1 + 2\lambda,\ 1 + \lambda$
d.r of $x + y + z + 1 = 0$ is 1, 1, 1
Both are at right angle $1 + 1 + 2\lambda + 1 + \lambda = 0 \Rightarrow \lambda = -1$
Required plane $\Rightarrow x - y + 5 = 0$

43. B

Sol. Equation of ellipse (E) is $\frac{(x-1)^2}{45} + \frac{(y-3)^2}{20} = 1$

P : $a^2 - b^2 = 25$

Q : Product of length of perpendicular from foci upon any tangent = $b^2 = 20$

R : Lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at P(4, 7) meet the normal PG and bisect it

$\therefore$ Required point is mid-point of PG
Equation of normal at P(4, 7) is 3x – y – 5 = 0

$\therefore\ G = \left(\frac{8}{3}, 3\right)$

$\therefore$ Required point = $\left(\frac{10}{3}, 5\right)$

S : Locus of mid-point of QR is another ellipse having same eccentricity as that of ellipse (E)

$\Rightarrow\ e = \frac{\sqrt{5}}{3}$

44. C

Sol. (P) $444\int_{-1}^{1}\frac{3x^{443}+x^{1331}+8x^{884}\sin x^{871}}{1+x^{888}}dx$

$= \int_{-1}^{1}\frac{444x^{443}(3+x^{888})}{1+x^{888}} + 444\int_{-1}^{1}\frac{8x^{884}\sin x^{871}}{1+x^{888}}dx$ (odd function)

Put $x^{444} = t$

$= \int_{-1}^{1}\left(\frac{3+t^2}{1+t^2}\right)dt$

$\Rightarrow I = \left[t + 2\tan^{-1}t\right]_{-1}^{1} = 2+\pi$

(Q) $I = \int_{0}^{2024} x\cos(2\pi\{x\})dx$

$I = \int_{0}^{2024}(2024-x)\cos 2\pi(1-\{x\})dx$

$2I = \int_{0}^{2024} 2024\cos(2\pi\{x\})dx = (2024)^2\left[\frac{\sin 2\pi x}{2\pi}\right]_0^1 = 0$

(R) $\frac{9}{10^4}\left(\sqrt{\frac{1}{10^4}}+\sqrt{\frac{2}{10^4}}+.....+\sqrt{\frac{10^4}{10^4}}\right) = 9\left(\frac{1}{10^4}\sum_{r=1}^{10^4}\sqrt{\frac{r}{10^4}}\right)$

$= 9\int_{0}^{1}\sqrt{x}dx = 6$

(S) $y = \int_{x}^{x^2}\frac{dt}{t+\sqrt{t}}$

$\frac{dy}{dx} = \frac{x+2\sqrt{x}-1}{(x+1)(x+\sqrt{x})} = 0 \Rightarrow x = 3-2\sqrt{2}$

$y = \int_{x}^{x^2}\frac{2dt}{2\sqrt{t}(\sqrt{t}+1)} = 2\left[\ln\sqrt{t}+1\right]_x^{x^2} = 2\ln\frac{x+1}{\sqrt{x}+1}$

Least value of $\frac{e^y}{(\sqrt{2}-1)^2} = 4$

45. D

Sol. (P) $3\sin^2 A + 2\sin^2 B = 1$

$\Rightarrow \cos 2A + \frac{2}{3}\cos 2B = 1$ ..... (1)

Also, $\frac{\sin 2A}{\sin 2B} = \frac{2}{3}$ putting this value in equation (1)

$\Rightarrow \sin(2A + 2B) = \sin 2B \Rightarrow 2\cos(A + 2B)\cdot\sin A = 0$

$\Rightarrow \cos(A + 2B) = 0$ (as $0 < A < \frac{\pi}{2}$)

$A + 2B = \frac{\pi}{2}$

(Q) $\frac{\cos 3x}{\sin 5x} - \frac{\sin 3x}{\cos 5x} = \frac{2\cos 8x}{\sin 10x}$

$= \frac{2(\cos 2x\cdot\cos 10x + \sin 2x\cdot\sin 10x)}{\sin 10x}$ = 2(sin 2x + cos 2x . cot 10x

(R) $\cos^{-1}(4x^3 - 3x) = \begin{cases} 3\cos^{-1}x & ; \frac{1}{2} < x < 1 \\ 2\pi - 3\cos^{-1}x & ; 0 < x < \frac{1}{2} \end{cases}$

$f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} & ; \frac{1}{2} < x < 1 \\ \frac{3}{\sqrt{1-x^2}} & ; 0 < x < \frac{1}{2} \end{cases} \Rightarrow \lambda = -2\sqrt{3}, \ \mu = 2\sqrt{3}$

(S) A, P, I, Q, R are concyclic

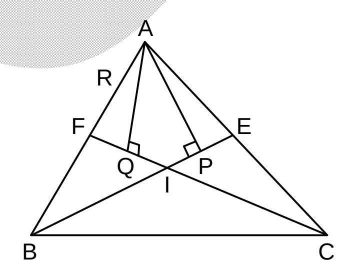
AI = Diameter

$\angle BIC = \frac{\pi}{2} + \frac{A}{2}$

Using sine rule

AR = AI sin ($\angle$AIR) and PQ = AI (sin$\angle$PAQ)

So, AR = PQ



SECTION – B

46. 743

Sol. Let $\left[x + \frac{19}{100}\right] + \left[x + \frac{20}{100}\right] = ... = \left[x + \frac{\lambda}{100}\right] = n$

and $\left[x + \frac{\lambda+1}{100}\right] = \left[x + \frac{\lambda+2}{100}\right] = ... = \left[x + \frac{91}{100}\right] = n + 1$

$\Rightarrow (\lambda - 18)n + (91 - \lambda)(n + 1) = 546$ (n, λ are natural number)

The only value satisfies about equation is n = 7, λ = 56

$\Rightarrow \left[x + \frac{56}{100}\right] = 7 \Rightarrow 644 \le 100x < 744$

$\Rightarrow \left[x + \frac{57}{100}\right] = 8 \Rightarrow 743 \le 100x < 843$

so $743 \le 100x < 744 \Rightarrow [100x] = 743$

47. 1

Sol. $ff(x) = \frac{(1 - x^{2011})^{\frac{1}{2011}}}{-x}$

fff(x) = x

$\therefore f_{2025}(x) = x = -\{x\}$ has one real solution

48. 1

Sol. $f(x) = x^{\frac{1}{x}} = e^{\frac{1}{x}\ln x}$

$f'(x) = x^{\frac{1}{x}}\left(\frac{1}{x^2} + \ln x\left(-\frac{1}{x^2}\right)\right) = x^{\frac{1}{x}}\left(\frac{1}{x^2} - \frac{\ln x}{x^2}\right) = \frac{x^{\frac{1}{x}}}{x^2}(1 - \ln x)$

f(x) is decreasing for x > e

$\Rightarrow$ 2026 > 2025 ; f(2026) < f(2025)

$\Rightarrow (2026)^{\frac{1}{2026}} < (2025)^{\frac{1}{2025}} \Rightarrow (2026)^{2025} < (2025)^{2026}$

$L = (2025)^{2026}\left[1 + \left(\frac{(2026)^{2025}}{(2025)^{2026}}\right)^n\right]^{\frac{1}{n}} = (2025)^{2026}$

a – b = –1

49. 5

Sol. Consider the expansion of $\left(\frac{2}{3} - \frac{1}{3}\right)\left(\frac{4}{5} - \frac{1}{5}\right)\left(\frac{6}{7} - \frac{1}{7}\right).....\left(\frac{2n}{2n+1} - \frac{1}{2n+1}\right)$ the negative terms corresponds to an odd number of tails. So product is (probability of even – probability of odd)

The product reduces to $\frac{1}{2n+1}$ obviously probability even + probability odd = 1

$\Rightarrow$ Probability odd = $\frac{1 - \frac{1}{2n+1}}{2} = \frac{n}{2n+1}$

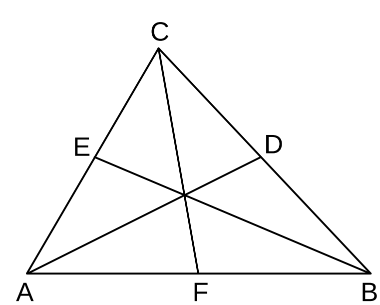
50. 61

Sol. Under given condition the possible mapping is f(–2) = 2, f(0) = 3, f(1) = 1

$\Rightarrow$ Area of Δ = $\frac{1}{2}\begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ -2 & 1 & 0 \end{vmatrix} = \frac{1}{2}\sqrt{61}$

51. 4

Sol. Let A be the origin then $\overrightarrow{AB} = \vec{c} \Rightarrow |\vec{c}| = 4$



$\overrightarrow{AC} = \vec{b} \Rightarrow |\vec{b}| = 3$

$|\overrightarrow{AD}| = \frac{4\vec{b} + 3\vec{c}}{7}$

Also, $\frac{\overline{AB}}{\overline{FB}} = \frac{3}{2} \Rightarrow \overrightarrow{AF} = \frac{3}{5}\vec{c}$ and $\overrightarrow{AE} = \frac{2}{3}\vec{b}$

Now, ΔABC = k(ΔDEF)

$\Rightarrow \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}k|\overrightarrow{DF} \times \overrightarrow{DE}|$

$\Rightarrow |\vec{c} \times \vec{b}| = k\left|\left(\frac{3}{5}\vec{c} - \left(\frac{4\vec{b} + 3\vec{c}}{7}\right)\right) \times \left(\frac{2}{3}\vec{b} - \left(\frac{4\vec{b} + 3\vec{c}}{7}\right)\right)\right| \Rightarrow k = \frac{245}{56}$