FIITJEE

ALL INDIA TEST SERIES

FULL TEST – VI

JEE (Main)-2024

TEST DATE: 18-01-2024

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 questions.

Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Physics

PART – A

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. The particle displacement of a travelling longitudinal wave in air is represented by s = f(x, t). The midpoints of compression zone and adjacent rarefaction zone are represented by letter C and R which of the following is incorrect.

(A) $\left|\frac{\partial s}{\partial x}\right|_C = \left|\frac{\partial s}{\partial x}\right|_R$

(B) $\left|\frac{\partial s}{\partial t}\right|_C = \left|\frac{\partial s}{\partial t}\right|_R = 0$

(C) $(\text{presure})_C - (\text{pressure})_R = 2\left|\frac{\partial s}{\partial x}\right|_C$ (Bulk modulus of air)

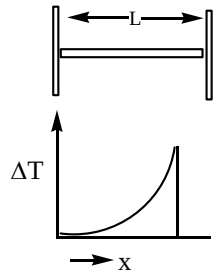
(D) Particles of air are stationary at mid way between C and R

2. A long conductor carries current i_0 along its length uniformly distributed over the cross sectional area. The magnetic energy inside the conductor per unit length is

(A) $\frac{\mu_0 i_0^2}{2\pi}$ (B) $\frac{\mu_0 i_0^2}{4\pi}$

(C) $\frac{\mu_0 i_0^2}{16\pi}$ (D) $\mu_0 i_0^2$

3. A steel bar of natural length 'L' is held between rigid supports and heated non uniformly in such a manner that the temperature increases ΔT at a distance x from one end is given by $\Delta T = \frac{T_0 x^2}{L^2}$ as shown in figure. The stress in the bar is (Assume that Young's modulus of elasticity for steel is E and thermal expansion coefficient is α)



(A) $\frac{E\alpha T_0}{3}$ (B) $\frac{E\alpha T_0}{9}$

(C) $3E\alpha T_0$ (D) $9E\alpha T_0$

4. A particle performs SHM on x-axis with amplitude A and time period T. The time taken by the particle to travel a distance $\frac{4A}{5}$ starting from rest is

(A) $\frac{T}{20}$ (B) $\frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$

(C) $\frac{T}{2\pi}\sin^{-1}\left(\frac{1}{5}\right)$ (D) $\frac{T}{2\pi}\cos^{-1}\left(\frac{1}{5}\right)$

5. A point source of light of power 'p' and wavelength 'λ' is emitting light in all directions. The number of photons present in a spherical region of radius r to radius 2r with centre at source is

(A) $\frac{p\lambda}{4\pi r^2 hc}$ (B) $\frac{p\lambda r}{hc^2}$

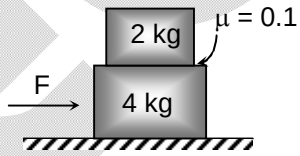
(C) $\frac{p\lambda}{4\pi r hc^2}$ (D) $4\pi\lambda r^2 hcp$

6. A disc of mass 'm' and radius R is given a linear velocity V_0 on a rough horizontal surface. Initially the motion of the disc is pure translatory. The disc sliding for sometime, attains pure rolling. The work done by friction in changing the motion of disc from pure translatory to pure rolling is

(A) Zero (B) $\frac{1}{3}mv_0^2$

(C) $\frac{1}{6}mv_0^2$ (D) $\frac{1}{12}mv_0^2$

7. The coefficient of state friction between two blocks of masses 2 kg and 4 kg is $\mu = 0.1$ and table is smooth. The maximum horizontal force 'F' needed to be applied on block of mass 4 kg that they move together is (g = 10 m/s^2)



(A) 6 N (B) 7 N
(C) 8 N (D) 9 N

8. What amount of heat will be generated in a coil of resistance R due to a charge 'q' passing through it if the current in the coil decreases down to zero uniformly during time interval Δt

(A) $\frac{4}{3}\frac{q^2R}{\Delta t}$ (B) $\frac{2}{3}\frac{q^2R}{\Delta t}$

(C) $\frac{3}{4}\frac{q^2R}{\Delta t}$ (D) $\frac{3}{2}\frac{q^2R}{\Delta t}$

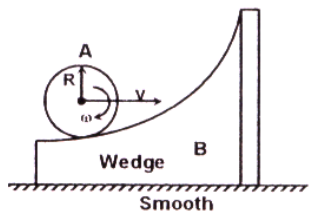
9. A capacitor of capacitance 'C' is connected in series with a resistor of resistance R. The combination is connected across a battery of emf E. The time in which potential difference across the capacitor becomes twice the potential difference across resistor is

(A) $RC\log_e 3$ (B) $RC\log_e 5$
(C) $RC\log_e 7$ (D) $RC\log_e 8$

10. An electric charge +q moves with a velocity $V = 3\hat{i} + 4\hat{j} + \hat{k}$ in an electromagnetic field given by $E = 3\hat{i} + \hat{j} + 2\hat{k}$, $B = \hat{i} + \hat{j} - 3\hat{k}$. The y – component of force experienced by +q is

(A) 2 q (B) 11 q
(C) 5 q (D) 3 q

11. In the figure shown, a cylinder A is initially rolling with velocity v on the horizontal surface of the wedge B (of same mass as (A). All surfaces are smooth and B has no initial velocity. Then maximum height reached by cylinder.



(A) $\frac{v^2}{4g}$ (B) $\frac{v^2}{g}$

(C) $\frac{v^2}{2g}$ (D) $\frac{v^2}{8g}$

12. A particle of mass 'm' is rotating in a circle of radius 'r' with power $P = mk^2r^2t$. The centripetal acceleration is

(A) kr^2t^2 (B) k^2r^2t

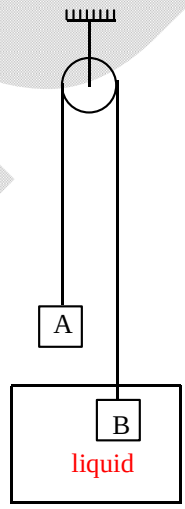
(C) k^2rt^2 (D) kr^2t

13. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to x-y plane) with its axis fixed about y-axis and free to rotate about its axis. The platform is given a motion in x-direction given by $x = A\cos\omega t$. There is no slipping between cylinder and platform. The maximum torque acting on the cylinder about centre of mass of cylinder during its motion is

(A) $M\omega^2AR$ (B) $\frac{2}{3}M\omega^2AR$

(C) $\frac{1}{3}M\omega^2AR$ (D) $\frac{3}{4}M\omega^2AR$

14. In the arrangement as shown $m_B = 3M$, density of liquid is ρ and density of block B is 2ρ. The system is released from rest so that block B moves up when in liquid and moves down when out of liquid with the same acceleration. The mass of block A is



(A) $\frac{7}{4}M$ (B) 2 M

(C) $\frac{9}{2}M$ (D) $\frac{9}{4}M$

15. The heat supplied to one mole of an ideal monoatomic gas in increasing the temperature from T_0 to $2T_0$ is $2RT_0$. The process in which the gas follows is

(A) PV = constant (B) $\frac{P}{V}$ = constant

(C) $\frac{V}{P}$ = constant (D) PV^2 = constant

16. A plane progressive wave given by y = A sin(kx $-\omega$t) is propagate in thin wire of linear mass density λ. Then which of the following is CORRECT at t = 0

(A) Net momentum of all the particle between x = 0 and x = λ is zero

(B) The magnitude of angular momentum of all the particles between O to $\frac{\lambda}{2}$ about origin (O) is same as that of the particle between $\left(\frac{\lambda}{2} \text{ to } \lambda\right)$

(C) The net angular momentum of all the particles between x = 0 to x = λ is zero about origin (O)

(D) All the particles are at their respective extremes at t = 0 so the speed is zero for all the particles.

17. A ring of moment of inertia 10^{-2} kgm^2 about an axis passing through its centre and perpendicular to its plane is placed with its centre at origin. The current is passed through ring so that magnetic moment $\vec{M} = (4\hat{i} - 3\hat{j})$ A-m^2. Magnetic field $\vec{B} = (3\hat{i} + 4\hat{j})$ tesla is switched on at t = 0. Which one of the following is correct option.

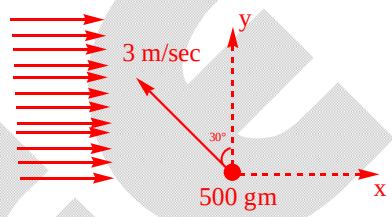
(A) Angular acceleration of ring at t = 0 is 5000 rad / sec^2

(B) Angular acceleration of ring at t = 0 is 3000 rad / sec^2

(C) Maximum angular velocity of the ring is 100 rad / sec^2

(D) Both (A) and (C)

18. Figure shows a sphere of mass 500 gm moving in a steady flow of air in the x-direction. The air steam exerts an essentially constant force of 0.9 N on the sphere in the x-direction. If at t = 0 the sphere is moving as shown in the figure. Determine the time 't' required for the sphere to cross the y-axis again 1.



(A) 1.66 sec (B) 2.66 sec

(C) 3.66 sec (D) 4.66 sec

19. The potential difference applied to an x-ray tube is increased. As a result in the emitted radiation

(A) Intensity decreases (B) Minimum wavelength increases

(C) Intensity remains unchanged (D) Minimum wavelength decreases

20. The separation between two coherent point sources S_1 and S_2 vibrating in phase emitting light of wavelength 'λ' is 2λ. The smallest distance from S_2 on a line passing through S_2 and perpendicular to S_1S_2 where a minimum intensity occurs is

(A) $\frac{2\lambda}{12}$ (B) $\frac{5\lambda}{12}$

(C) $\frac{7\lambda}{12}$ (D) $\frac{11\lambda}{12}$

SECTION – B

(Numerical Answer Type)

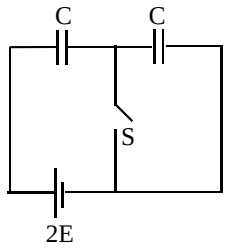
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. A planet revolves around the sun in an elliptical orbit of semi major axis 2×10^{12} m. The areal velocity of the planet when it is nearest to sun is 4.4×10^{16} m^2 / sec. The least distance between planet and sun is 1.8×10^{12} m. The minimum speed of planet in km/sec is 10x. The value of x is

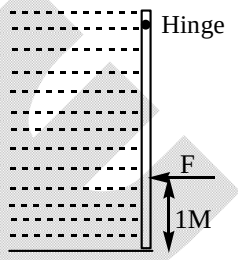
22. The charges $\frac{q}{2}$ and 2q are placed at the centre of face and corner of a cube respectively. Total flux through the cube is $\frac{nq}{2\varepsilon_0}$. The value of 'n' is

23. A radioactive sample decays through two different decay processes α and β dacay. Half life time for α decay is 3 hr. and half life time for β decay is 6 hr. Find the ratio of number of initial radioactive nuclei to the number of radioactive nuclei present after 6 hr.

24. Switch 'S' is open for long time and then closed. The heat generated in the circuit due to switching is $\frac{CE^2}{P}$. The value of P is



25. A square gate of size (4 m $\times$ 4 m) is hinged at top most point. A liquid of density ρ fills the space left of it. The force acting at a height of 1 m from lowest point can hold gate stationary is $\frac{2^n}{9}\rho g$. The value of n is

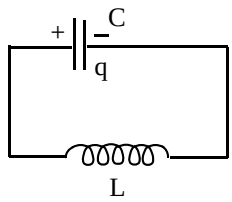


26. When a wave pulse is propagating then displacement of a particle 'x' at time t = 0 and t = 2 sec are given by Ae^{-5x^2} and $Ae^{-5x^2+40x-80}$. Speed of pulse is given by 2p m/sec. Value of p is

27. A target element A is bombarded with electrons and the wavelengths of the characteristic spectrum are measured. A second characteristic spectrum is also obtained, because of an impurity in the target. The wavelength of the K_α lines are 196 pm (element A) and 169 pm (impurity). Find the atomic number of impurity. (atomic number of element A is 27).

28. A spherical balloon is filled with an ideal mono-atomic gas. At some instant its pressure was P_0 volume V_0 and temperature T_0. If the balloon expanded in such a manner so that the rate of radiation from the balloon remained constant. It was found that the work done by gas inside the balloon was $6P_0V_0\left[1-\frac{1}{\sqrt{n}}\right]$, when its volume increased to $8V_0$. Find the value of n. (assume ambient temperature ot be close to 0K)

29. In the L-C circuit as shown, C = 1 F, L = 11 H. Charge in the capacitor is 4 coulombs at time t = 0 and it is decreasing at the rate of $\sqrt{3}$ A. The maximum charge on the capacitor is n coulomb. Find n.



30. Two wheels each marked with a dot on its rim are mounted side by side. When the wheels are at rest, the dots are aligned. One wheel is given a constant angular acceleration of $\frac{\pi}{2}\text{rad/sec}^2$ and the other is given a constant angular acceleration of $\frac{\pi}{4}\text{rad/sec}^2$. The time after which the two dots will be aligned again for the first time (in seconds) is

Chemistry

PART – B

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

31. Which of the following statements is correct about metal complexes with d^8 configuration?
(A) This configuration is ideal for the formation of high spin square planar complexes with strong field ligands in Nickel (II) complexes.
(B) With bulky ligands like triphenyl phosphine, metal generally prefer tetrahedral high spin complexes as there will be more steric hindrance in tetrahedral complexes than in square planar complex.
(C) $\left[Ni(H_2O)_6\right]^{2+}$, $\left[Ni(NH_3)_6\right]^{2+}$ and $\left[Ni(en)_3\right]^{2+}$, all complexes are paramagnetic with single unpaired electron.
(D) For $\left[Ni(PPh_2Et)_2Br_2\right]$ complex, an equilibrium mixture of two geometries exist in solution.

32. Which of the following options is **correct**?
(A) The oxidizing power of the permanganate is strongly pH dependent.
(B) The thermodynamic instability for a particular oxidation state of manganese in its compound does not link with its disproportionation.
(C) The factor responsible for the Lanthanoid contraction, i.e. the imperfect shielding of one electron by another in the same set of orbitals, does not observed in an ordinary transition series.
(D) The almost identical radii of Zr (160 pm) and Hf (159 pm) is a consequence of the actinoid contraction.

33. Disodium dihydrogenpyrophosphate is mixed with sodium bicarbonate and used in bread making to leaven the bread, that is to make it rise. They react together and evolve a gas when heated together. Which gas it is
(A) H_2O_2 vapour (B) CO_2
(C) P_2O_5 (D) P_4O_{10}

34. In which of the following reactions O_2 gas is one of the products?

I. $KClO_3 \xrightarrow[MnO_2(catalyst)]{150^oC} Products$

II. $HOCl \xrightarrow[\text{catalytic decomposition}]{Co^{2+}} Products$

III. $CO(g) + \underset{(steam)}{H_2O(g)} \xrightarrow[FeCrO_4]{673\ K} Products$

IV. $LiNO_3(s) \xrightarrow{\Delta} Products$

V. $NaHCO_3 \xrightarrow{100^oC} Products$

VI. $NH_4NO_3 \xrightarrow{\Delta} Products$

VII. $NH_4NO_2 \xrightarrow{\Delta} Products$

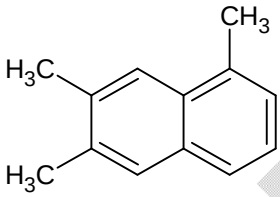
(A) I, II and VI (B) I, II, III and IV
(C) I, II and IV (D) II, III and IV

35. How many different A – O (A being any element, O-oxygen atom) bond length is recorded in all the below molecules/compound?
Na_2SO_3, $NaHSO_3$, $NaHSO_4$, SO_3, O_3, O_2, KO_2, P_4O_{10}
(A) 10 (B) 12
(C) 13 (D) 14

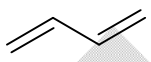
36. Which of the following does not contain six membered cyclic rings?
(A) Retinol (B) Limonene
(C) Menthol (D) Geraniol

37. Methylene is an example of carbene type intermediate. It has two electronic states one with two unbonded electrons present in a single sp^2 hybridized orbital, called as singlet carbene.
The another has two unbonded electrons present in two unhybridised p orbitals with carbon having sp hybridization, called as triplet carbene.
Based on the following two statements, choose the correct option.
Statement I: The triple carbene with the two unbonded electrons having same spins is more stable than the triplet carbene with the two unbonded electrons having opposite spins.
Statement II: Hund's rule of maxmum multiplicity states that if two or more orbitals of equal energy are available, electrons will occupy them with same spins.
(A) Both Statements are correct and Statement II is the correct explanation of Statement I.
(B) Both Statements are correct and Statement II is not the correct explanation of Statement I.
(C) Statement I is true, Statement II is false.
(D) Statement I is false, Statement II is true.

38. The IUPAC nomenclature of the following molecules:



(A) 1, 6, 7-Trimethylnaphthalene (B) 2, 3, 5-Trimethylnaphthalene
(C) 1, 3, 4-Trimethylnaphthalene (D) 2, 3, 4-Trimethylnaphthalene

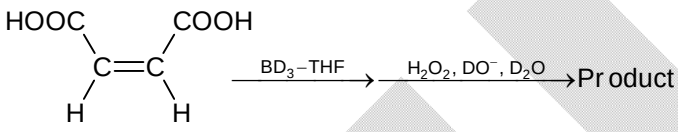
39.  $+ Br_2\,(1\,mole) \longrightarrow A + B$
If the product's mixture contain two structural isomer groups A and B.
Which of the following statement is correct?
(A) If A is chiral, then it will be the major product at higher temperature.
(B) If A is chiral, then it will be the minor product at higher temperature.
(C) At lower temperature the product's mixture shows more optical activity than at higher temperature.
(D) In actual, there are total 2 products formed.

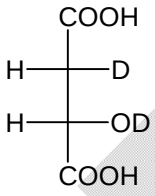
40. **Statement I:** H_2 reacts more faster with Cl_2 than D_2.
Statement II: H_2 has a lower energy of activation than D_2.
(A) Both statements are correct, statement II is the correct explanation of statement I.
(B) Both statements are correct, statement II does not explain statement I.
(C) Statement I is correct, statement II is incorrect.
(D) Both statements are incorrect.

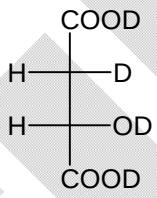
41. Which of the following is the correct option?
(A) Introduction of electron withdrawing groups into the o – or p – positions of the diazonium cation further reduces its electrophilic character.
(B) Coupling reaction is found to be acid-catalysed only.
(C) The difference in position of attack of electrophile on mono-substituted benzene probably does not reflects the relative electron density of the various positions.
(D) Diazonium cation is a weak electrophile compared to nitronium cation.

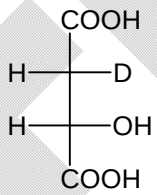
42. The products obtained at cathode and anode made up of platinum by the electrolysis of aqueous solution of cupric nitrate is:
(A) Hydrogen and Nitric oxide
(B) Hydrogen and Oxygen
(C) Copper, Oxygen
(D) Copper and Nitric oxide

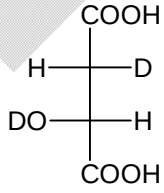
43. For fairly concentrated solution of a weak electrolyte A_xB_y, having concentration 'C' the degree of dissociation is given by:
(A) $\alpha = \sqrt{k_{eq} xy / c}$
(B) $\alpha = \sqrt{K_{eq} c / xy}$
(C) $\alpha = \sqrt{K_{eq} / c(x+y)}$
(D) $\alpha = \left[K_{eq} / \left(c^{x+y-1}.x^x.y^y\right)\right]^{1/x+y}$

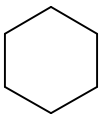
44. 

(A) 

(B) 

(C) 

(D) 

45.  $\xrightarrow[h\nu]{Br_2}$ Dibrominated product(s)

The total number of dibrominated product(s)

(A) 9 (B) 6

(C) 4 (D) 7

46. Diethylether $\xrightarrow[2.\ Zn/Ether,\ \Delta]{1.\ PCl_5,\Delta}$ A $\xrightarrow[(monobromination)]{Br_2,\ h\nu}$ $\underset{Major}{B} + \underset{Minor}{C}$

$B \xrightarrow{NaOH(aq.)} D \xrightarrow{PCC} E$

$C \xrightarrow{NaOH(aq.)} F \xrightarrow{PCC} G$

The correct option is

(A) A is 2-methylpropane

(B) E and G can be distinguished by Tollen's reagent

(C) D and F can be distinguished by reaction with PDC

(D) C is 2-Bromobutane

47. $BCl_3 + H_2O \longrightarrow \underset{\text{Weak acid}}{A} + \underset{\text{Strong acid}}{B}$

$A + NaOH \rightleftharpoons C$

The hybridization of the central atom of C.

(A) sp^3 (B) sp^2

(C) sp^3d (D) sp^3d^2

48. A mixture of Mn_3O_4 and Al (in ratio 3 : 8 by mole) is used in thermite reduction method for the extraction of Mn. Find the heat evolved (kJ) per gram of the mixture

Given: $\Delta_f H^o\left(Al_2O_3\right) = -1700\ kJ\ mol^{-1}$

$\Delta_f H^o\left(Mn_3O_4\right) = -950\ kJ\ mol^{-1}$

Molar mass of Al, O and Mn are 27, 16 and 55 g mol^{-1} respectively.

(A) 10 (B) 4.37

(C) 58.90 (D) 16.42

49. Colour of manganese borate salt in oxidizing flame during borax bead test is

(A) Red (B) Violet

(C) Blue (D) Green

50. Which of the following reactions does not involve a nucleophilic addition step?

(A) Acid catalysed esterification of carboxylic acid and alcohol.

(B) Aldol condensation reaction

(C) Reduction of carboxylic acid through $LiAlH_4$

(D) Reaction of phenol with bromine water.

SECTION – B

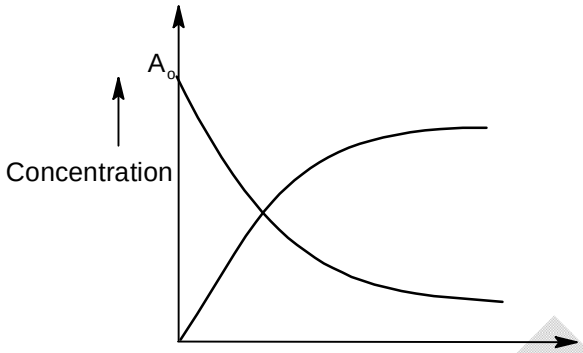
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

51. The initial concentration of reactant A in the given reaction is A_o

$2A \longrightarrow 3B$

The concentration of B, at the point of intersection of the two curves shown, is N times of A_o. Find the value of 10N.



52. The uncertainty in the position and velocity of a gaseous molecule are 4.960×10^{-6} m and 10^{-4} m / s, respectively.

The gaseous molecules has same root mean square velocity at 300 K as that of oxygen gaseous molecule at T K temperature. Find the value of T. $\left(h = 6.62 \times 10^{-34}\ \text{Jsec},\ \pi = 3.14\right)$

53. $CH_3COCH_3 + HC \equiv CH \xrightarrow{KOH} (A) \xrightarrow[Pd-BaSO_4 \text{ sulphur}]{H_2} (B) \xrightarrow{Al_2O_3, \Delta} (C)$

Natural rubber is a polymer of C. The molecular mass of B(g mol^{-1}) is

54. 1 mole of CO_2 gas along with some amount of CaO is present in a cylindrical vessel fitted with movable piston on one side at 300 K. Initially the volume of the vessel is 20 litre. How much work (in Joule) should be done to compress the vessel reversibly and isothermally till $CaCO_3$ first appears?

$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g), \qquad K_p = 6.15 \text{ atm (at 300 K)}$

$\left[\text{Given}: R = 0.082 \text{ atm Lt mol}^{-1} \text{ k}^{-1}; \ln 5 = 1.6\right]$

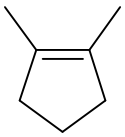
55. In an aq. solution containing KCl and KI each having 0.01 M concentration, aq. $AgNO_3$ is added slowly. The concentration of I^- ion in that solution when AgCl starts precipitating is $N \times 10^{-9}$ M. Find the value of N.

[Given : $K_{sp}(AgCl) = 2 \times 10^{-10}$; $K_{sp}(AgI) = 8 \times 10^{-17}$]

56. 1 mole of glucose is added into 2000 g of water. The freezing point of pure water is $0^\circ C$. The temperature of the solution is brought down to $-1.1625^\circ C$. Find the percentage of water which comes out in the form of ice. $\left(K_f = 1.86 \text{ k kg mol}^{-1}\right)$

57. The standard electrode potential of $I_2 / 2I^-$ is 0.619 V. Find the equilibrium constant for the reaction $I_2 + I^- \longrightarrow I_3^-$. The standard electrode potential of $I_3^- / 3I^-$ is 0.5355 V.

(Given: $\frac{2.303RT}{F} = 0.059, \log 676 = 2.83$)

58. 

$$\xrightarrow[\text{cold}]{\text{dil. } KMnO_4} (A) \xrightarrow{HIO_4} (B) \xrightarrow[\Delta]{OH^-} (C)$$

(P)

How much larger is the molecular mass of C than P?

59. How many number of correct statements:

1. Cellulose is more easily hydrolysed than starch.
2. Complete hydrolysis of cellulose by heating with dilute H_2SO_4 under pressure yields β-D-(+)-glucose only.
3. Amylose is the water soluble component of starch.
4. Amylose is an unbranched chain of α-D-glucose units connected together y α-1,6-glycosidic linakage.
5. The main chain in amylopectin consists of an α-1,4-glycosidic linkage.
6. The branches in amylopectin are connected to the main chain by α-1,4-glycosidic linkage.
7. When glucose is allowed to react with one mole of phenyl hydrazine, glucose phenyl hyrazone is formed which is insoluble in water.
8. Glucose reduces ammoniacal $AgNO_3$ solution to produce metallic silver and itself get oxidized to gluconic acid..
9. Dextron is a condensation polymer, derived from the monomers glycolic acid and lactic acid.

60. A mixture of cyclohex-2-enone, pentan-3-one, 2-methylcyclohexanone, 2,6-dimethylcyclohexanone and pentan-2-one is dissolved in a solution of D_2O containing DCl. Find the maximum number hydrogen atoms that can be exchanged for deuterium. (Avoid making addition product).

Mathematics

PART – C

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. The area of region(s) enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$

(C) $\frac{1}{3}$ (D) 1

62. If y = f(x) satisfies the condition f′(x) sinx + f(x) cosx = 1 , f(x) being bounded when $x \to 0$. If $I = \int_0^{\pi/2} f(x)\,dx$, then

(A) $\frac{\pi}{2} < I < \frac{\pi^2}{4}$ (B) $\frac{\pi}{4} < I < \frac{\pi^2}{2}$

(C) $1 < I < \frac{\pi}{2}$ (D) $0 < I < 1$

63. The sum of infinite terms of the series $\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)$ + ... is equal to

(A) $\tan^{-1}(1)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}(3)$ (D) $\tan^{-1}(4)$

64. If $\sum_{i=1}^{6}\left(\sin^{-1}(x_i)+\cos^{-1}(y_i)\right) = 9\pi$, then $\int_{\sum_{i=1}^{6} x_i}^{\sum_{i=1}^{6} y_i} x\ln\left(1+x^2\right)\left(\frac{e^x}{1+e^{2x}}\right)dx$ is

(A) 0 (B) $e^6 + e^{-6}$

(C) $\ln\left(\frac{37}{12}\right)$ (D) $e^6 - e^{-6}$

65. If $\int \frac{dx}{x^{2008}+x} = \frac{1}{p}\ln\left(\frac{x^q}{1+x^r}\right)$ + C, where p, q, r ∈ N and need not to be distinct, then the value of p + q + r is

(A) 6024 (B) 6022

(C) 6021 (D) 6020

66. Let $f(\theta)=\frac{1}{1+(\cot\theta)^x}$ and $S=\sum_{\theta=1^\circ}^{89^\circ} f(\theta)$, then [S] is. (where [.] denotes greatest integer function)

(A) 44 (B) 45
(C) 46 (D) 43

67. Let $\vec{r}=(\vec{a}\times\vec{b})\sin x+(\vec{b}\times\vec{c})\cos y+2(\vec{c}\times\vec{a})$, where $\vec{a},\vec{b},\vec{c}$ are non-zero and non-coplanar vectors. If $\vec{r}$ is orthogonal to $\vec{a}+\vec{b}+\vec{c}$, then the minimum value of $\frac{20}{\pi^2}(x^2+y^2)$, is

(A) 25 (B) 30
(C) 70 (D) 15

68. If 'O' be on interior point of ΔABC such that $2(\overrightarrow{OA})+5(\overrightarrow{OB})+10(\overrightarrow{OC})=\vec{O}$. If the ratio of the area of ΔABC to the area of ΔAOC is 't', where 'O' is the origin. Then [t] is (where [.] denotes greatest integer function)

(A) 5 (B) 6
(C) 3 (D) 7

69. The point of intersection of the plane $\vec{r}\cdot(3\hat{i}-5\hat{j}+2\hat{k})=6$ with the straight line passing through the origin and perpendicular to the plane $2x - y - z = 4$, is (x_0, y_0, z_0). The value of $2x_0 - 3y_0 + z_0$, is

(A) – 1 (B) – 2
(C) 4 (D) 2

70. Equation of image of the line $x + y = \sin^{-1}(a^6 + 1) + \cos^{-1}(a^4 + 1) - \tan^{-1}(a^2 + 1)$, $a \in R$ about x-axis is given by

(A) $x - y = 0$ (B) $x-y=\frac{\pi}{2}$
(C) $x - y = \pi$ (D) $x-y=\frac{\pi}{4}$

71. If x, y, z be positive numbers such that $x + y + z = 10$. If maximum value of $x^2y^3z^5$ is N and number of odd divisors is 'k' then $\frac{k}{12}$ is

(A) 12 (B) 6
(C) 8 (D) 2

72. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on the set A = {1, 2, 3} is

(A) reflexive but not symmetric (B) reflexive but not transitive
(C) symmetric and transitive (D) neither symmetric nor transitive

73. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $\det(A^n - I) = 1 - \lambda^n$, $n \in N$ then the value of λ is

(A) 1 (B) 2
(C) 3 (D) 4

74. The matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ be a zero divisor of the polynomial $f(x) = x^2 - 4x - 5$. If 'm' be the trace of matrix 'A^3', then $\left[\frac{m}{20}\right]$, is (where [.] denotes greatest integer function)

(A) 6 (B) 5
(C) 7 (D) 4

75. If 'z' be a complex number satisfying $\left(z+\frac{1}{z}\right)\left(z+\frac{1}{z}+1\right) = 1$. Then the value of $\left(3z^{100}+\frac{2}{z^{100}}+1\right)\left(z^{100}+\frac{2}{z^{100}}+3\right)$

(A) 36 (B) 6
(C) 9 (D) 8

76. The complex number 'z' satisfying the equations $|z| = 1$ and $\left|\frac{z-\sqrt{2}(1+i)}{z}\right| = 1$, is

(A) $1 - i$ (B) $\frac{1+i}{\sqrt{2}}$

(C) $\frac{1-i}{\sqrt{2}}$ (D) $-i$

77. If $f(x) = \lim\limits_{n\to\infty} \frac{\sin\left(\pi x^4\right) + (x+2)^n \frac{\tan(\pi x)}{(x+1)}}{1+(x+2)^n - x^4}$, and $\lim\limits_{x\to -1} f(x) = m$, then $\left[\frac{m}{\pi}\right]$, is (where [.] denotes greatest integer function)

(A) $-\pi$ (B) π
(C) 0 (D) 1

78. If m and n be the co-efficient of x^3 in $(1 + x + 2x^2 + 3x^3)^3$ and $(1 + x + 2x^2 + 3x^3 + 4x^4)^3$, respectively then $\left[\frac{m}{n}\right]$, is (where [.] denotes greatest integer function)

(A) -1 (B) 1
(C) 2 (D) -2

79. If $a^2 + 4b^2 + 9c^2 + 2ab + 6bc + 3ac \le 0$ where $a, b, c \in R$, then the number of distinct real roots of $ax^2 + bx + c = 0$ is/are

(A) 0 (B) 2
(C) 1 (D) more than 2

80. If α and β be real numbers and $f(x) = \alpha \sin x + \beta \sqrt[3]{x} + 4$, $\forall x \in R$. If $f\left(\log_{10}\left(\log_3 10\right)\right) = 5$, then $f\left(\log_{10}\left(\log_{10} 3\right)\right)$, is

(A) 3 (B) -3
(C) 2 (D) -2

SECTION – B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

81. If the mean deviation about median for the numbers 3, 5, 7, 2k, 12, 16, 21, 24 arranged in ascending order is 6 (given $\frac{7}{2} < k < 6$) , then the median is _____

82. If $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$, are collinear and α abc + β(a + b + c) = γ(ab + bc + ca), where α, β, $\gamma \in$ N and in their lowest form. Then the value of $[\alpha+\beta+\gamma]$ is _____ (where [.] denotes greatest integer function)

83. A ray of light moving parallel to the x-axis get reflected from a parabolic mirror $(y-2)^2 = 4(x+1)$. Then the point on the axis of the parabola through which the ray must pass after reflection is (0, k), where $k \in$ N. Then k is _____

84. If P is any point on ellipse with foci S_1 and S_2 and eccentricity is 1/2, such that $\angle PS_1S_2 = \alpha$, $\angle PS_2S_1 = \beta$, $\angle S_1PS_2 = \gamma$ and $\frac{\cot\left(\frac{\alpha}{2}\right)+\cot\left(\frac{\beta}{2}\right)}{\cot\left(\frac{\gamma}{2}\right)} = \lambda$, where $\lambda \in$ N, then λ is _____

85. For each positive integer n, consider the point P with abscissa n on the curve $y^2 - x^2 = 1$. If d_n represents the shortest distance from the point P on the line y = x. Then $\lim\limits_{n\to\infty}(n \cdot d_n)$ has the value equal to $\frac{1}{k\sqrt{k}}$, where $k \in$ N then k is ____

86. Let $S_1 : x^2 + y^2 + 4x - 6y - 12 = 0$, and $S_2 : x^2 + y^2 - 3x - 2y + 1 = 0$ be two distinct circles with centres C_1 and C_2 respectively. If A and B are two points which lie on their common chord as well as common tangents and $C_1A^2 - C_2B^2 = \frac{a}{b}$, where a, b $\in$ N and a, b are co-prime. Then a + b is_____

87. A number 'p' is selected from the set of first 9 natural numbers (i.e. p = 1, 2, …, 9). If the probability that f(f(p)) = p, where $f(p) = p^2 - 3p + 3$ is k/9, then k is _____

88. A and B are two independent events whose probabilities are respectively $\frac{1}{n}$ and $\frac{1}{n+1}$. If $P(A \cap B) = \frac{1}{12}$, then 'n' is _____

89. If the distance of a point 2i from the line $(1 + i)z - (1 - i)\bar{z} + 2i = 0$ (where $i = \sqrt{-1}$) is 'd'. then $\sqrt{2}d$ is _____

90. If $4l^2 - 5m^2 + 6l + 1 = 0$ and lx + my + 1 = 0 touches a fixed circle having radius $\sqrt{k}$, then k is_____